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AN EXPLORATORY APPROACH TO ADVANCED MATHEMATICS

ABSTRACT. There is little current use of more exploratory approaches to the teaching and learning of advanced mathematics. This article describes a simple exploratory teaching model, and reports the findings of a small-scale action-research project to implement and evaluate it. There are discussions of student perceptions and attitudes, and of student attainment. The principal findings are that there was no underachievement under the exploratory approach, and that student preferences for routinised mathematical activity and directed teaching style were strongly associated.

BACKGROUND TO THE RESEARCH

In recent years mathematics teaching has been much influenced by the Cockcroft Report (1982); in particular by the statement that

Mathematics teaching at all levels should include opportunities for
- exposition by the teacher;
- discussion between teacher and pupils and between pupils themselves;
- appropriate practical work;
- consolidation and practice of fundamental skills and routines;
- problem solving, including the application of mathematics to everyday situations;
- investigational work.

In 1982, when the Report appeared, the regular use of this range of teaching strategies could already be found in some classrooms. In particular, pedagogical theories emphasising exploratory play, practical work, and discussion, were well developed, if not then widely practised, at primary level. Similarly, this style of working was not unknown at lower secondary level, and has become more widespread in recent years in the wake of corresponding curriculum reforms.

At upper secondary level, however, there was little practice on which to build. In a survey published in the same year as the Report, Her Majesty's Inspectors of Schools (1982) – HMI – describe the classroom processes which predominate at that level.

The general style of teaching in the sixth form closely resembles the approach adopted with fourth and fifth year pupils.... The teacher presents a topic on the blackboard, works through an example and while the students carry out exercises based on the topic, the teacher helps individuals.... The majority of lessons visited ... were mainly instructional in character, made only limited provision for the interchange of ideas with students and offered insufficient opportunity for students to gain more than a restricted view of the subject. Teaching in these circumstances was highly predictable and tailored to the needs of what had

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become a passive and uncritical audience.... The teaching seen during the survey relied predominantly on training students in a sequence of techniques, each illustrated by a comparatively limited range of stereotyped exercises.

AN EXPLORATORY TEACHING MODEL

The goal of the action research which this article describes was to implement and evaluate a more exploratory teaching approach to advanced mathematics. The major difference between the conventional and experimental approaches can best be understood in terms of two very simple models of the sequencing of classroom processes, shown in Figure 1. Whereas in the conventional approach new ideas are introduced through teacher exposition, in the experimental approach teacher exposition is deferred, and draws on prior student investigation.

Within the conventional model, of course, exposition can take many forms; lecture, question and answer, class discussion. What unites these forms, nonetheless, is the central and consistent role of the teacher in directing the development of new ideas.

In the experimental model, introduction to new ideas is through a two phase process in which exploration – a form of investigation – is followed by codification – a form of exposition. In the exploration phase students investigate relatively open problem situations. These are chosen to reactivate what Husserl (1978) describes as the 'primary origins' of mathematical concepts, and to encourage the 'improvement of guesses by speculation and criticism' which Lakatos (1976) identifies as the heart of the mathematical process. During this phase, the role of the teacher is to stimulate and

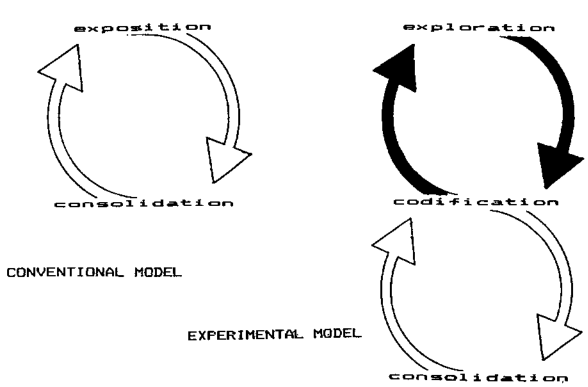

Fig. 1

support exploration, and to gather and analyse information about the strategies and theories being employed by students. In the codification phase the teacher draws on this evidence in developing the powerful ideas and strategies of institutionalised mathematics. Through interactive exposition, the teacher shapes students' experience of the problem situations, and relates their ideas and strategies to those of institutionalised mathematics.

It may be helpful to give an example of the model in practice; in particular of the kind of activities used in the exploration phase, and the way in which these are drawn on in the codification phase. One exploratory activity, used at an early stage with both experimental classes in the study, concerned the factorisation of polynomials.

A short introduction was necessary to explain the activity to the students. First they were reminded of the idea of a rectangle number and asked to arrange 12 cubes in different configurations – including one 'in three dimensions' – to illustrate the factorisations of the number 12. It was then explained that the objective of the activity was to look at the extension of the idea of factorisation to algebraic expressions, which could be thought of as generalised number expressions. This involved the use of multibase apparatus, shown in Figure 2.

A base-4 'long' was introduced to represent a letter such as 'x'. Students were asked to construct some simple linear polynomials such as $x + 2$ and $2x + 3$ using longs and cubes. A 'flat' was then produced and students were asked to say what it would represent. Some answers of '$4x$' were forthcoming. A base-10 long and flat were shown, and students were reminded that their interpretation had to fit any value of x, not just 4. They were now happy to describe a flat as x^2, and could apply the same reasoning to correctly describe a 'block' as x^3.

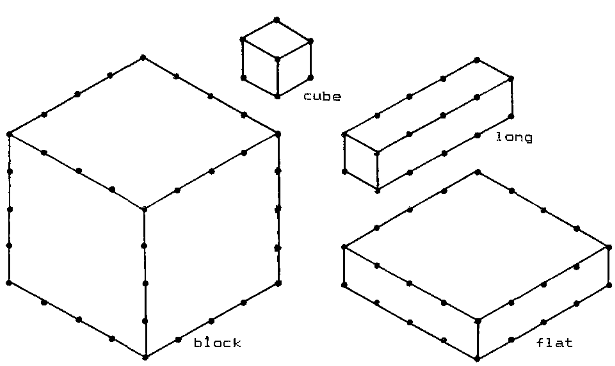

Fig. 2

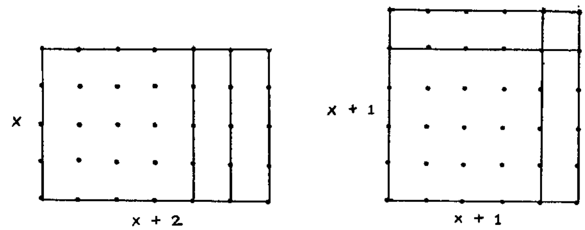

Fig. 3

Finally, the students were shown an ungrouped representation of $x^2 + 2x$, and asked first to configure it as a rectangle (as shown in Figure 3a), and then to interpret this configuration as a factorisation. The outcomes were recorded on square spotty paper, and the factorisation checked by expanding. The polynomials $x^2 + 2x + 1$ (as shown in Figure 3b) and $x^2 + 2x + 2$ were similarly investigated by the class as a whole: not all the students were convinced that the latter could not be factorised.

Now familiar with the conventions and purpose of the activity, the students were asked to work with the materials in groups. Their task was to examine four sets of polynomials (shown in Table I) carefully chosen to encourage speculation about pattern and to introduce progressively greater complexity.

One group of students initially produced configurations dependent on a particular base (as shown in Figure 4a): although they realised that $x^2 + 5x + 4$ was neither $x(x + 6)$ nor $4(x + 6)$ they were unable to resolve the conflict on their own. Nor was all recording accurate at first (illustrated by Figure 4b). But, asked to repeat the tasks with base-10 material, the students gained a deeper insight into the underlying problem. They came to appreciate the importance of choosing a configuration independent of a particular value of x, and the value of seeking more highly schematised configurations (illustrated by Figure 4c). The principles underlying schematisation were drawn on at the codification stage in seeking a general relationship between a quadratic polynomial and its factorisation.

Several groups noted that one member of the second set seemed to have

TABLE I

x^2+5x+4	$2x^2+6x+6$	x^3+2x^2+x	x^2-1
x^2+5x+5	$2x^2+7x+6$	x^3+2x^2+x+1	x^2+1
x^2+5x+6	$2x^2+8x+6$	x^3+2x^2+x+2	x^3+1
			x^3-1

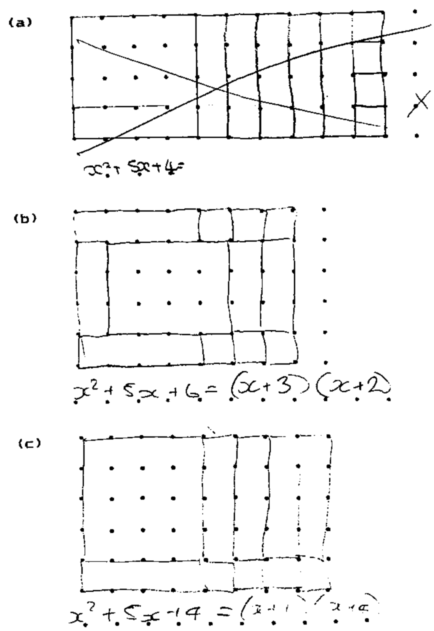

Fig. 4

more than one factorisation. Groups were encouraged to exchange information and to look for a connection between the different factorisations. Three distinct factorisations were found (as shown in Figure 5). At the codification stage the relationship between these factorisations was analysed. Comparison of these factorisations with those of the number 12 was used to draw out various relationships of prime numbers and irreducible polynomials. This helped to clarify the conventional idea of uniqueness of factorisation.

All groups initially thought that the third member of the cubic set could not be factorised. Asked to revise the idea that the configuration must form a cuboid, a factorisation (shown in Figure 6b) was found which passed quickly between groups, to much initial scepticism, but eventual enlightenment. This was drawn on at the codification stage in discussing the factorisation patterns of cubic polynomials.

There was speculation not just about particular factorisations, but about more general patterns. For example, in the light of evidence from x^2+2x+2 and x^2+5x+5, one group proposed that any polynomial of the form x^2+kx+k could not be factorised. It was suggested that they test x^2+4x+4. Back came the modified conjecture that k must not be a proper square, in response to which x^2+9x+9 was suggested.

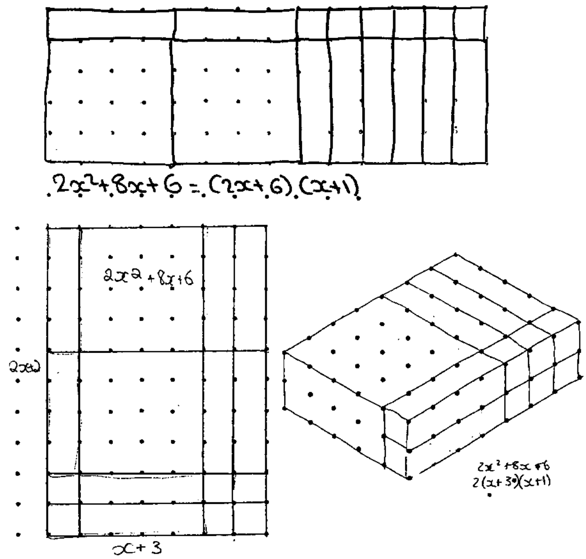
2x²+8x+6 = (2x+6)(x+1)
2x²+8x+6
2x+2
x+3
2x²+8x+6
2(x+3)(x+1)

Fig. 5

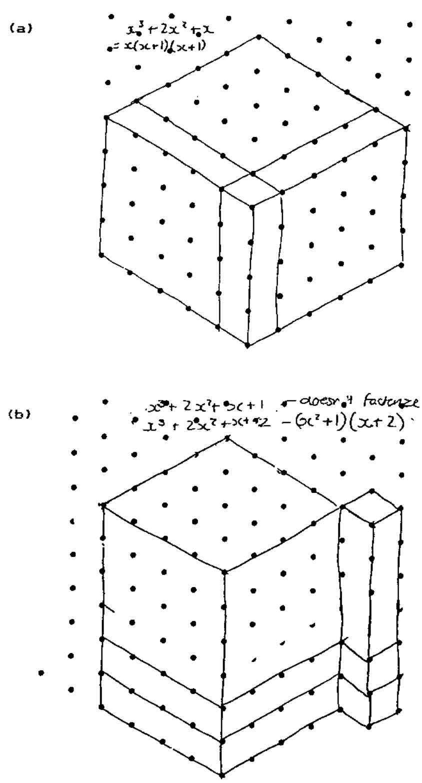
(a)
x³+2x²+x
= x(x+1)(x+1)
(b)
x³+2x²+x+1 ← doesn't factorize
x³+2x²+x+2 = (x²+1)(x+2)

Fig. 6

Although this lesson involved every student in some elements of modelling, schematising, abstracting and generalising, the activity was, of course, not the same for all of them. Some already had a good grasp of quadratic factorisation at a symbolic level, whereas others had little or none. Some started from a conception of the letter x as a particular value, while for others it was already a generalised number. Clearly such aspects influenced students' patterns of thinking. But one purpose of the activity was precisely to encourage students to articulate and explore different ideas and approaches. Equally, students found their challenge at different levels: for some, the first problem set of quadratics was a struggle; for others the activity seemed trivial until they tackled the cubic and decomposition problem sets.

IMPLEMENTING THE TEACHING MODEL

For three mornings a week during the 1987/88 academic year, I was seconded to a local sixth-form college with a reputation for lively and effective teaching of a conventional style, producing excellent examination results. The mathematics department had several part-time teachers, and I became one of these, teaching within the normal departmental framework. The students I taught belonged to the last cohort to take O-level examinations and were in the first year of an A-level mathematics course. They had not, then, been exposed to the reformed mathematics curriculum of the new GCSE examinations. In the mathematics department, every set had two teachers, each responsible for half of the syllabus. I taught two contrasting sets.

The first contained 15 students studying for a single A-level in Mathematics: all but 2 were female; their O-level attainments in Mathematics were relatively low; and none were studying physics at A-level. I taught this set Algebra & Geometry and Statistics for two sessions – each of a little over 1 hour – every week. The second set contained 14 students, studying for A-levels in Mathematics and Further Mathematics: all but 2 were male; their O-level attainments in Mathematics were relatively high; all but one had also studied for an Additional Mathematics O-level; and 10 were studying physics at A-level. I taught this set Algebra & Geometry and Mechanics for three sessions every week.

Very briefly, over the course of the academic year I taught the following topics. In Algebra & Geometry both sets studied surds, indices and logarithms; polynomial algebra (concluding with the factor and binomial theorems); sequences and series (concluding with the theory of arithmetic and geometric progressions); arrangements and selections (concluding with the theory of permutations and combinations); spatial trigonometry (in two and three dimensions); algebraic trigonometry (concluding with the great variety of standard two-variable identities): the double-subject set also studied complex numbers (concluding with de Moivre's theorem); and vector geometry (concluding with points, lines and planes in three dimensions). In Statistics, the single-subject set studied measures of centre and spread (concluding with their algebraic properties); basic probability (concluding with conditional probability and independence); and discrete random variables (concluding with the theory of uniform, binomial and geometric distributions). In Mechanics, the double-subject set studied the mechanics of particles (and systems of particles) under forces of variable magnitude but fixed direction (including gravitational, frictional and elastic forces) using concepts of force, momentum and energy; and particle kinematics (concluding with the motion of projectiles).

To teach this not inconsiderable material in a limited time represents a considerable challenge. Indeed, many teachers perceive coverage of extensive syllabus content within limited time as a major constraint on teaching style. In particular, a more exploratory style is seen as too time consuming. I was very aware of these pressures, but found that it was possible, by careful planning, to teach in an exploratory style while maintaining the rate of coverage required by the departmental syllabuses – reflecting in turn the demands of examination syllabuses.

Essentially this entailed spending much of the class contact time on exploration and codification of new ideas, with the majority of consolidation carried out in private study time – an established and important feature of study at this level. Equally homework was regularly set and marked to ensure effective feedback, but students were encouraged to refer first to texts and printed solution sheets, rather than expecting every aspect of their homework to be gone over in class.

From my point of view, a more serious constraint was the lack of conceptual emphasis in the prescribed work. A great deal of mathematical technique was expected of students, but relatively little understanding of why and where particular techniques worked or did not work. The syllabuses, in common with others at this level, emphasised a collection of isolated skills, without strong unifying concepts and principles.

A second constraint frequently cited by teachers is the reluctance of students to engage in more exploratory styles of working; to use practical materials, for example, or to engage in group discussion. Again, I understood the potential problems. My response was to establish an exploratory style of working from the students' first encounter with me. In particular, I was relieved to find that, as long as they could seem some purpose to the activity, they were willing – indeed often pleased – to work with cuisenaire rods and multibase apparatus; cardboard cutouts and acetate transparencies; constructional shapes; dice, cards, counters, balls and coins.

There was, however, some difference in response to group work. The single-subject set took very readily to this, and I was impressed by the quality of discussion, and the effectiveness of collaboration, between students in their groups of three or four. In the double-subject set, more competitiveness was evident at first, and I found that students generally worked most effectively together in pairs, although two isolate students – one highly independent-minded, another quiet and lacking confidence – were rarely able to pair effectively with other students. On extended project work, however, the members of this set were generally more ready to work in larger groups, and to exchange ideas.

From the teacher's point of view, a more exploratory style calls for particular pedagogic skills: giving students the opportunity to think for themselves, but providing appropriate stimulus or support where necessary; helping groups to work constructively together, and individuals to participate effectively in groups; gathering ideas from students and identifying critical issues for clarification at the later codification stage.

STUDENT PERCEPTIONS AND ATTITUDES DESCRIBED

Towards the end of the first term, students in the experimental classes were invited, but not required, to complete an unstructured questionnaire on their experience of studying mathematics that term: in particular, they were asked to comment on any important contrast with their previous experience of studying mathematics. About half the students chose to respond.

Virtually all the respondents commented that mathematics was now much more concerned with explanation and justification. It was clear that for many students this was in marked contrast to their previous experience of learning mathematics. Typical comments suggested that, while previously,

> [you were] told what to do and how to do it with no explanation of why it worked,

> instead of just accepting . . . you now have to prove that it is true and find out why. You don't just accept it, you have to discuss it and reason it out.

Some students also compared the two strands of their current mathematics course in similar terms:

> Work is done in greater depth; . . . things are proven, investigated further, and generalised. This is especially true [in the exploratory teaching style].

[In the conventional style] one has to make a mental jump from something which looks totally irrelevant... to something pretty obscure... and I can not make that jump... and have to be led by the hand, just as in O-level, which is pretty humiliating.

During the second term, a student teacher with an interest in group-work observed sequences of lessons over several weeks, and interviewed students in the experimental classes. He reported that students found the exploratory approach more lively and stimulating than that met in other lessons. The students also considered that it created a more informal classroom atmosphere in which they were less inhibited about admitting lack of understanding. Student opinion, however, was divided on whether learning was more successful. A common observation was that this approach made more demands on students as it required them to think and discuss far more than during conventional lessons.

During the third term, as internal examinations approached, it was natural that the demands of examinations should start to preoccupy students. At the end of this term, immediately following the internal examinations, the mathematics department undertook an annual exercise in which all students were expected to complete a structured questionnaire, commenting frankly on the teaching that they had received, and in particular making suggestions to their two teachers as to how it might be improved. I had access to the comments of the students in the experimental sets, who had experienced both conventional and exploratory teaching approaches. For each set the questionnaires were administered by my teacher colleague. Because of absences at the end of term, four students in the single-subject set and one in the double-subject set did not complete this questionnaire.

In the questionnaire, students made separate comments on the methods employed by their two teachers. The rubric asked students, remembering the constraints on time during lessons, to assess which of several types of activity should be given more or less time in class to help their learning. Students responded on a five point scale from 'a lot less time' to 'a lot more time'. As expected, there was little use of the two extreme points on the scale, their main function being to encourage students to deviate from the centre of the response range.

The four items which seemed to particularly capture the contrasting views of students were: time for teacher explanation of a new topic; time for the teacher to do worked examples; time to discover rather than being told; time to research a topic on your own. In these first two activities it is the teacher who takes the initiative in identifying and shaping ideas, whereas the latter two give more opportunity and responsibility to students.

Together, responses to these items can be thought of as indicating whether students sought more or less teacher direction in relation to each of the two styles of teaching that they had experienced.

A crude index was computed by scoring each item response as 1 if it sought more teacher direction, 0 for no change, -1 for less teacher direction; then aggregating the four scores to produce a change rating for each student in relation to each teaching style. Thus the extreme rating of 4 for a particular teaching style would indicate that the student wishes more time to be given to teacher explanation of new topics and to teacher demonstration of worked examples, and less to student discovery and individual research; whereas the opposite extreme of -4 would indicate exactly the reverse pattern of preferences.

It must be emphasised that this index yields two scores which indicate the change that a student would like to see in each style. And while we may reasonably infer that a student who gives different change ratings to the two styles perceives them as different (in a way which we can interpret), the converse – that a student who perceives the styles as different will give them different ratings – is (as we shall see) not always true. The scattergrams of change rating pairs for the two experimental groups are shown in Figure 7.

This data supports the underlying premise that students were experiencing different teaching styles. The visual pattern suggests that students discriminated clearly and appropriately between the two teaching styles, as

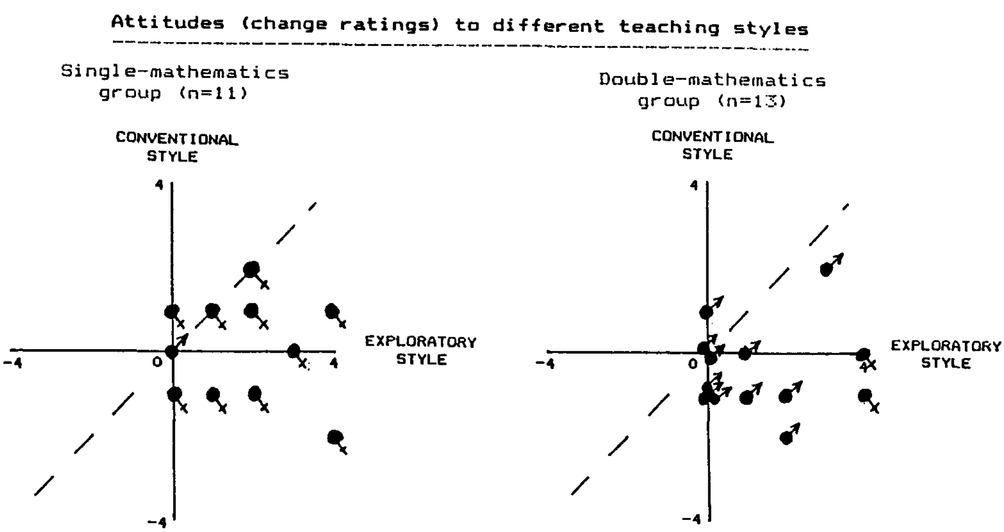

Fig. 7

the great majority of points – those lying below the broken line – reflect a perception of the exploratory style as less teacher directed than the conventional style. Further support comes from written comments by students which explicitly contrast the two teaching styles.

The combination of teachers I had . . . was balanced – one was stimulating and the other was more traditional and slightly more informative.

More formally, the statistical results shown in Table II indicate that the change ratings of the exploratory style differed from those of the conventional style, in the anticipated direction. On the appropriate directed t-test, the difference in change ratings is significant beyond the 5% level for the single-subject set, and beyond the 1% level for the double-subject set.

These results are even stronger than they appear. Several students sought little or no change in either style, but included comments which indicated that they perceived the styles as different. One such student, for example, commented,

I like having two teachers with very different approaches to teaching.

Thus difference in change ratings is likely to underestimate perceived difference.

Further analysis of the pattern of comments and scores suggests that the students can fruitfully be divided into three categories. The 'flexible' student, located close to the origin of the scattergram, is at ease with both the conventional and exploratory approaches, and recommends little change in the style of either teacher. The 'compromise' student, lying out in the lower right quadrant, seeks an intermediate, more balanced approach, with ratings of both the conventional and experimental styles suggesting a

TABLE II

Attitudes (change ratings) to different teaching styles

	Exploratory style	Conventional style	Matched differences
Single-mathematics group ($n = 11$)			
mean	1.73	0.09	1.64
deviation	1.49	1.22	2.01
t-score			2.70
Double-mathematics group ($n = 13$)			
mean	1.31	−0.38	1.69
deviation	1.55	1.04	1.80
t-score			3.39

move towards the other. The 'directed' student, lying out in the upper right quadrant, is attracted to a strongly teacher directed approach.

After careful scrutiny of borderline cases, 'flexible' was operationalised in terms of neither rating being of magnitude greater than 1; 'compromise' as the remainder of the lower right quadrant, excluding the horizontal axis; 'directed' as the remainder of the upper right quadrant, including the horizontal axis. The single-mathematics group contained 5 'flexible' students, 2 'compromise' students, and 4 'directed' students; the double-mathematics group, 8 'flexible' students, 3 'compromise' students, and 2 'directed' students.

The 18 students in the 'flexible' and 'compromise' groups had much in common: in particular, they recognised positive qualities in both the exploratory and conventional styles. By contrast, the 'directed' students were clearly ill at ease with the exploratory style: indeed, 4 of the 6 students in this category sought more teacher direction not only of the exploratory style but of the conventional style.

STUDENT PERCEPTIONS AND ATTITUDES THEORISED

What differentiates the 'directed' students from their peers? And how might these differences be explained? To me, both as ~~a~~ teacher and as a researcher, these seem to be important questions. In this section I will move beyond the immediate evidence to attempt to identify key underlying relationships. This speculation will, however, remain rooted in observation.

In the post-examination questionnaire, the written comments of the 'directed' students placed a strong emphasis on the value of worked examples provided by the teacher to illustrate a standard method, and of practice by students of a range of similar examples. A typical comment was,

> The more worked examples I'm shown and I do, the better I remember and understand maths

It was not only the 'directed' students who made considerable use of worked examples in revising for examinations: the knowledge that most of the questions would be of standard types encouraged this strategy. But for the 'directed' students this was more than simply a strategy of examination preparation; this was what learning mathematics was all about.

Discussion with students confirmed other evidence that, for most of them, this kind of approach had underpinned their previous experience of learning mathematics. But whereas the 'flexible' and 'compromise' groups were attracted to certain features of an approach which emphasised thinking things out for oneself from first principles, the 'directed' group strongly preferred a more prescriptive approach based on routinised mathematical activity and authoritative mathematical knowledge.

This desire for authoritive mathematical knowledge was clear in other comments.

I would get on better [with the exploratory style if there were] definite formulas .... A lot of this part of the course seems to be assumptions I've made.

I think it would be easier to be told the formula first, and then if the teachers think it would be helpful for us to deduce our own formula at least we would know what we were aiming for.

In discussion, the students who made these comments referred to a critical classroom incident. They had been doing exploratory work on arithmetic sequences, physically modelling them with cuisenaire rods, and had arrived at a formula for the sum of the first n members of such a sequence in terms of the first member, a, and the common difference, d: $na + \frac{1}{2}n(n-1)d$. In the codification phase, arguments to support this result were written down more formally, and I mentioned to the students that there was another version of the formula in their textbooks: $\frac{1}{2}n(2a + (n-1)d)$. They showed by algebraic manipulation that the two versions were equivalent. At the time, one student had asked which was the 'correct' formula, and I had replied that, since they were equivalent, both were 'correct'. Yet these students clearly continued to be concerned that the formula that they had developed was not the 'official' one, and this – and other similar incidents – had unsettled them.

One of the most strongly 'directed' students advocated a highly prescriptive style of teaching which left little scope for choice or initiative on the part of students: an emphasis on learning through worked examples; a view of exploratory work as quite interesting but time consuming and unproductive; a preference for notes to be given by the teacher rather than compiled by the student; and an emphasis on tests and 'neat' homeworks to force students to work. This highly instrumental perspective was, however, exceptional in the experimental sets, although colleagues reported that it was not an uncommon phenomenon.

To examine some of these issues in a more formal way a number of potentially significant variables were operationalised in terms of available data. A variable, DIRECTION, was formed by adding the separate change ratings for the exploratory and conventional teaching styles to give a single score for each student, which discriminated well between the 'flexible' and 'compromise' groups on the one hand, and the 'directed' group on the other. Gender was clearly potentially important. A variable, GENDER, was coded -1 for female, 1 for male. Equally, there were considerable differences, not only in gender, but in prior attainment, between single –

and double-subject groups. A variable, LEVEL, was coded -1 for single-subject, 1 for double-subject.

The post-examination questionnaire included a number of items in which students were asked to appraise themselves by rating personal statements such as 'I spend sufficient time on my homework' on a five point scale from 'strongly agree' to 'strongly disagree'. Considerations of face validity and statistical association were used to group some of these items to give three further variables indicating how students rated themselves on aspects of studying mathematics: COMPETENCE ('I usually understand the work', 'I find the work easy', 'I usually know the answers when asked'); APPLICATION ('I concentrate well during class discussion', 'I spend sufficient time on my homework', 'I give work in on time'); and INITIATIVE ('I find quick methods', 'I persevere with problems').

Because of the predominance of female students in the single-subject group and of male students in the double-subject group, there was potential for confusion between GENDER and LEVEL effects. Inspection of the full and partial correlations between variables established, however, that GENDER was a much stronger variable. In particular, the correlation between GENDER and DIRECTION changes little, and remains significant at the 1% level, when the influence of LEVEL is removed. In comparison, the association of LEVEL with DIRECTION is both less strong and less stable. In theoretical terms, too, GENDER is a more powerful explanatory variable.

A number of important observations can be drawn from the full and partial correlations in Table III. First, it is clear that male students tend to

TABLE III

Association (correlation) between DIRECTION and GENDER and student self-ratings

	GENDER	COMPETENCE	APPLICATION	INITIATIVE
Full correlation				
DIRECTION	−0.49*	−0.08	−0.22	−0.62***
GENDER		0.44	0.38	0.58**
Partial correlation				
DIRECTION with GENDER removed		0.17	−0.04	−0.47†

† significant just below 1% level.
* significant at 1% level.
** significant at 0.5% level.
*** significant at 0.1% level.

rate themselves more highly on all traits; this echoes many other findings about differences in confidence between male and female groups. Second, female students tend to prefer strong teacher direction; female students tend to rate themselves particularly low on initiative and perseverance in problem solving; students who rate themselves low on initiative and perseverance in problem solving tend to prefer strong teacher direction. This identifies a key negative association between initiative and perseverance in problem solving and preference for strong teacher direction. Finally, students' self-ratings of their mathematical competence and their application to work do not seem to be strongly associated with their preferences in teaching style.

This quantitative evidence supports the earlier argument from more qualitative data: that the students who prefer a more directed teaching style are those who prefer routinised mathematical activity. Expressed in such bald terms, the statement sounds tautological, the argument circular. But there is more here than consistency of analysis. What is significant is that learned behaviour seems to play an important role in 'directed' students' preference for routinised mathematical activity; in particular the strong emphasis on routine in their experience of mathematics learning. Moreover, the tendency of female students to prefer routinised activity and greater teacher direction can be explained in terms of gender stereotypes which emphasise the desirability of rule-following rather than rule-challenging behaviour in girls (Walden and Walkerdine, 1986).

STUDENT ATTAINMENT EVALUATED

In discussions with teachers, including my colleagues at the sixth-form college, one major concern recurred: that adoption of a more exploratory teaching style may prejudice examination attainment, particularly among lower attaining students. Equally, HMI (1982) found this to be a widespread concern among sixth-form teachers. The conjecture behind this concern might be phrased more precisely as follows: that students taught in an exploratory style may underachieve in those aspects of mathematics which present examinations are designed to assess.

To throw some light on this matter, the examination attainment of students taught in exploratory style was compared with that of students of similar characteristics taught in conventional style. In selecting students for comparison, three potentially important factors were considered: gender; previous attainment in mathematics; other A-level subjects being studied concurrently. Previous attainment was measured in terms of O-level grades in Mathematics – and where relevant Additional Mathematics. Subject groupings were divided into three classes: physical science (physics and chemistry); biological science (biology and chemistry); humanities and social sciences (drawn from economics geography, social biology, history, languages, literature, art).

Finding a comparison group for the double-subject set was relatively easy, as there were two parallel sets. This permitted the adoption of a matched subjects design, reducing the influence of extraneous factors. Regrettably, a similar design was not possible with the single-subject set. Here, there was only one other set of students of similar previous attainment, and the students in this set were predominantly male, and predominantly studying physical science – in direct contrast, and, in the case of the latter factor, at potential disadvantage, to the experimental set. Here an unmatched subjects design was adopted.

The measures of attainment employed were based on the college internal, end-of-year examinations, closely modelled on the external examinations which the students would take after a further year of study. Comparisons of attainment in Algebra & Geometry were made for both single and double-subject sets. The Mechanics attainment of the double-subject set was also compared: a similar comparison for the single-subject set in Statistics was not possible, as the parallel set was studying Mechanics. To avoid the possibility of researcher bias, I was not involved in setting or marking the papers on which comparisons were made.

The results were clear cut. There was no difference in the attainments of students taught in exploratory and conventional styles. All three t-scores on the relevant tests were close to 0, and there was no trend in the directions. These results establish that there was no underachievement under the exploratory style.

So far, the discussion of attainment has been in aggregate terms, across the sample as a whole. But this may mask differential effects within the sample. In particular, attitudes to teaching style may be associated with differential attainment under differing styles. To investigate this issue, the mean standardised attainments of students under exploratory and conventional teaching styles were compared for each attitude subgroup. These are shown in Table IV.

None of the differences approaches significance, suggesting that attitude to teaching style is not linked to relative attainment under different teaching styles. In particular, these results offer evidence that 'directed' students do not underachieve under the exploratory style. Equally – and consistently – there was no evidence of female underachievement.

TABLE IV

Attainment (mean standardised scores) under different teaching styles: by attitude subgroup

	Exploratory style	Conventional style	Differences
Flexible ($n = 13$)	−0.10	−0.11	0.01
Compromise ($n = 5$)	0.41	0.36	0.05
Directed ($n = 6$)	0.15	0.23	−0.08

CONCLUSION

The principal findings of this study are that it is feasible to implement a more exploratory approach to the teaching of advanced mathematics; that such an approach produces no underachievement on conventionally measured outcomes, either in general, or on the part of particular attitude and gender subgroups; and that, despite previous experience, the majority of students adapt well to such an approach. Although limited in scale, and in variety of context, the research is based on a sustained experiment, conducted in a natural setting, with rigorous evaluation.

Another finding of interest is the strong association between preferences for routinised mathematical activity and for directed teaching style, and the particular prevalence of these preferences among female students.

This small-scale piece of action research was conducted without external funding or support. Potentially important future areas of research include systematic observation and analysis of classroom processes; the development of 'unconventional' measures to assess attainment in areas such as problem-solving and investigation, communication and interpretation; and further study of students' perceptions of, and attitudes to, the processes of teaching and learning.

REFERENCES

Cockcroft, W. H. (ch.): 1982, *Mathematics Counts: Report of the Committee of Enquiry into the Teaching of Mathematics in Schools*, DES, London.

Her Majesty's Inspectorate (HMI): 1982, *Mathematics in the Sixth Form*, DES, London.

Husserl, E.: 1978, 'The origin of geometry', in Luckmann, T. (ed.), *Phenomenology and Sociology*, Penguin, Harmondsworth.

Lakatos, I.: 1976, *Proofs and Refutations*: *The Logic of Mathematical Discovery*, Cambridge University Press, Cambridge.

Walden, R. and Walkerdine, V.: 1986, 'Characteristics, views and relationships in the classroom', in Burton, L. (ed.): *Girls Into Maths Can Go*, Holt, Rinehart and Winston, London.

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