



What Two ...? Extension Ideas

Well here's a question with just one answer - or so it would seem.

BUT what a wonderful opportunity for the pupils to learn about opening out such a question. This is done really by just asking, "I wonder what would happen if ...?"

A/ ... we took each pair in turn?

Well here are the answers you get if they decided to do that. But some might want to do the usual thing of exploring differences and digital roots - the second table shows some of what happens.

			first		Ans	Diff	D. R.
0	1	0	0	1	0		
1	2	2	1	2	2	2	2
2	3	6	2	3	6	4	6
3	4	12	3	4	12	6	3
4	5	20	4	5	20	8	2
5	6	30	5	6	30	10	3
6	7	42	6	7	42	12	6
7	8	56	7	8	56	14	2
8	9	72	8	9	72	16	9
9	10	90	9	10	90	18	9
10	11	110	10	11	110	20	2
11	12	132	11	12	132	22	6
12	13	156	12	13	156	24	3
13	14	182	13	14	182	26	2
14	15	210	14	15	210	28	3
15	16	240	15	16	240	30	6
16	17	272	16	17	272	32	2
17	18	306	17	18	306	34	9
18	19	342	18	19	342	36	9
19	20	380	19	20	380	38	2
20	21	420	20	21	420	40	6
21	22	462	21	22	462	42	3
22	23	506	22	23	506	44	2
23	24	552	23	24	552	46	3
24	25	600	24	25	600	48	6
25	26	650	25	26	650	50	2
26	27	702	26	27	702	52	9
27	28	756	27	28	756	54	9
28	29	812	28	29	812	56	2
29	30	870	29	30	870	58	6
30	31	930	30	31	930	60	3
31	32	992	31	32	992	62	2
32	33	1056	32	33	1056	64	3
33	34	1122	33	34	1122	66	6

The pupils can then be asked what things they notice.

B / ... the rule about consecutives was changed to mean two more on [or 3, 4, 5 etc.]?

So they could explore 1×3 , 2×4 , 3×5 ... etc.

	plus2	ans	D. R.		plus3	ans	D. R.		plus4	ans	D. R.
0	2	0	9	0	3	0	9	0	4	0	9
1	3	3	3	1	4	4	4	1	5	5	5
2	4	8	8	2	5	10	1	2	6	12	3
3	5	15	6	3	6	18	9	3	7	21	3
4	6	24	6	4	7	28	1	4	8	32	5
5	7	35	8	5	8	40	4	5	9	45	9
6	8	48	3	6	9	54	9	6	10	60	6
7	9	63	9	7	10	70	7	7	11	77	5
8	10	80	8	8	11	88	7	8	12	96	6
9	11	99	9	9	12	108	9	9	13	117	9
10	12	120	3	10	13	130	4	10	14	140	5
11	13	143	8	11	14	154	1	11	15	165	3
12	14	168	6	12	15	180	9	12	16	192	3
13	15	195	6	13	16	208	1	13	17	221	5
14	16	224	8	14	17	238	4	14	18	252	9
15	17	255	3	15	18	270	9	15	19	285	6
16	18	288	9	16	19	304	7	16	20	320	5
17	19	323	8	17	20	340	7	17	21	357	6
18	20	360	9	18	21	378	9	18	22	396	9
19	21	399	3	19	22	418	4	19	23	437	5
20	22	440	8	20	23	460	1	20	24	480	3
21	23	483	6	21	24	504	9	21	25	525	3
22	24	528	6	22	25	550	1	22	26	572	5
23	25	575	8	23	26	598	4	23	27	621	9
24	26	624	3	24	27	648	9	24	28	672	6
25	27	675	9	25	28	700	7	25	29	725	5
26	28	728	8	26	29	754	7	26	30	780	6
27	29	783	9	27	30	810	9	27	31	837	9
28	30	840	3	28	31	868	4	28	32	896	5
29	31	899	8	29	32	928	1	29	33	957	3
30	32	960	6	30	33	990	9	30	34	1020	3
31	33	1023	6	31	34	1054	1	31	35	1085	5
32	34	1088	8	32	35	1120	4	32	36	1152	9
33	35	1155	3	33	36	1188	9	33	37	1221	6

Again, by asking the pupils what they notice, further explorations are likely to occur.

Many pupils get fascinated by digital roots so they may go on to explore these further.

We see here what happens when we add on different amounts and just see the digital roots. And then - as someone noticed - rearrange the figures so that they follow a similar patterning:

plus 0	plus 1	plus 2	plus 3	plus 4	plus 5	plus 6	plus 7	plus 8	plus 9	plus 10	plus 11
1	2	3	4	5	6	7	8	9	1	2	3
4	6	8	1	3	5	7	9	2	4	6	8
9	3	6	9	3	6	9	3	6	9	3	6
7	2	6	1	5	9	4	8	3	7	2	6
7	3	8	4	9	5	1	6	2	7	3	8
9	6	3	9	6	3	9	6	3	9	6	3
4	2	9	7	5	3	1	8	6	4	2	9
1	9	8	7	6	5	4	3	2	1	9	8
9	9	9	9	9	9	9	9	9	9	9	9
1	3	9	1	6	6	1	9	3	1	3	9
4	6	3	4	9	9	4	3	6	4	6	3
9	2	8	9	5	5	9	8	2	9	2	8
7	9	6	7	3	3	7	6	9	7	9	6
7	9	6	7	3	3	7	6	9	7	9	6
9	2	8	9	5	5	9	8	2	9	2	8
4	6	3	4	9	9	4	3	6	4	6	3
1	3	9	1	6	6	1	9	3	1	3	9
9	2	8	9	5	5	9	8	2	9	2	8

C/ ... we were to look at the multiplication table to see the same thing?

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

The grey is consecutive adding on 1 each time, yellow adding 2, green adding 3 etc.

Again ask the pupils "What do you see?"

Strange things do occur like adding vertically from the dark blue to the green gives the following totals: 120, 180, 250, 330, 420 etc. What a surprise and why??

D/ ... we use 3 or 4 or 5 consecutive numbers together to do the multiplying?

	3consec	D.R.		4consec	D.R.		5consec	D.R.		6consec	D.R.
0	0	9	0	0	9	0	0	9	0	0	9
1	6	6	1	24	6	1	120	3	1	720	9
2	24	6	2	120	3	2	720	9	2	5040	9
3	60	6	3	360	9	3	2520	9	3	20160	9
4	120	3	4	840	3	4	6720	6	4	60480	9
5	210	3	5	1680	6	5	15120	9	5	151200	9
6	336	3	6	3024	9	6	30240	9	6	332640	9
7	504	9	7	5040	9	7	55440	9	7	665280	9
8	720	9	8	7920	9	8	95040	9	8	1235520	9
9	990	9	9	11880	9	9	154440	9	9	2162160	9
10	1320	6	10	17160	6	10	240240	3	10	3603600	9
11	1716	6	11	24024	3	11	360360	9	11	5765760	9
12	2184	6	12	32760	9	12	524160	9	12	8910720	9
13	2730	3	13	43680	3	13	742560	6	13	13366080	9
14	3360	3	14	57120	6	14	1028160	9	14	19535040	9
15	4080	3	15	73440	9	15	1395360	9	15	27907200	9
16	4896	9	16	93024	9	16	1860480	9	16	39070080	9
17	5814	9	17	116280	9	17	2441880	9	17	53721360	9
18	6840	9	18	143640	9	18	3160080	9	18	72681840	9
19	7980	6	19	175560	6	19	4037880	3	19	96909120	9
20	9240	6	20	212520	3	20	5100480	9	20	127512000	9
21	10626	6	21	255024	9	21	6375600	9	21	165765600	9
22	12144	3	22	303600	3	22	7893600	6	22	213127200	9
23	13800	3	23	358800	6	23	9687600	9	23	271252800	9
24	15600	3	24	421200	9	24	11793600	9	24	342014400	9
25	17550	9	25	491400	9	25	14250600	9	25	427518000	9
26	19656	9	26	570024	9	26	17100720	9	26	530122320	9
27	21924	9	27	657720	9	27	20389320	9	27	652458240	9
28	24360	6	28	755160	6	28	24165120	3	28	797448960	9
29	26970	6	29	863040	3	29	28480320	9	29	968330880	9
30	29760	6	30	982080	9	30	33390720	9	30	1168675200	9

E/ ... we were to add instead of multiply?

	2consec	D.R.	3consec	D.R.	4consec	D.R.	5consec	D.R.	6consec	D.R.
0	1	1	3	3	6	6	10	1	15	6
1	3	3	6	6	10	1	15	6	21	3
2	5	5	9	9	14	5	20	2	27	9
3	7	7	12	3	18	9	25	7	33	6
4	9	9	15	6	22	4	30	3	39	3
5	11	2	18	9	26	8	35	8	45	9
6	13	4	21	3	30	3	40	4	51	6
7	15	6	24	6	34	7	45	9	57	3
8	17	8	27	9	38	2	50	5	63	9
9	19	1	30	3	42	6	55	1	69	6
10	21	3	33	6	46	1	60	6	75	3
11	23	5	36	9	50	5	65	2	81	9
12	25	7	39	3	54	9	70	7	87	6
13	27	9	42	6	58	4	75	3	93	3
14	29	2	45	9	62	8	80	8	99	9
15	31	4	48	3	66	3	85	4	105	6
16	33	6	51	6	70	7	90	9	111	3
17	35	8	54	9	74	2	95	5	117	9
18	37	1	57	3	78	6	100	1	123	6

And so on. . . .

In order that the pupils can start asking these kinds of questions themselves and then explore further they will need to know it's OK to do so and feel encouraged that they could even follow something that you, the adult, has not thought of or seen before.