Stage 3 ★
Mixed Selection 3 – Solutions

1. Parallel base

Since ST is parallel to UV, $\angle PRT$ and the angle of size $132°$ are corresponding angles, so $\angle PRT = 132°$.

Since angles on a straight line sum to $180°$ we must have $\angle PRQ = 48°$.

By the exterior angle of a triangle theorem, $\angle SQP = \angle QPR + \angle PRQ$, so $134 = x + 48$, that is, $x = 86$.

2. Other side

The angles in triangle ACD must add up to $180°$, so $\angle CDA = 180° - 65° - 50° = 65°$.

This means that $\angle ACD = \angle CDA$, so ACD is an isosceles triangle. Therefore, $AC = AD$.

We know from the question that, $AD = BC$, so $BC = AC$. This makes ABC an isosceles triangle, and $\angle CAB = \angle ABC$.

Then $\angle ABC = \frac{1}{2}(180° - \angle ACB) = \frac{1}{2}(180° - 70°) = 55°$.

3. Half past two

In moving from one number on the clock face to the next, a hand moves $360° \div 12° = 30°$.

At $2{:}30$ the hour hand will be exactly half way between the 2 and the 3, and the minute hand will be exactly on the 6. So the angle between the two hands will be $3 \times 30° + 15° = 105°$

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)

4. **Tent poles**

As QS=SR, triangle SQR is isosceles, so $\angle SRQ = \angle SQR = x°$. So by the exterior angle theorem $\angle QST = 2x°$.

Also, $\angle TQS = 2x$ since $QT = TS$. As $PT = QT$, $\angle TPQ = \angle TQP = 20°$.
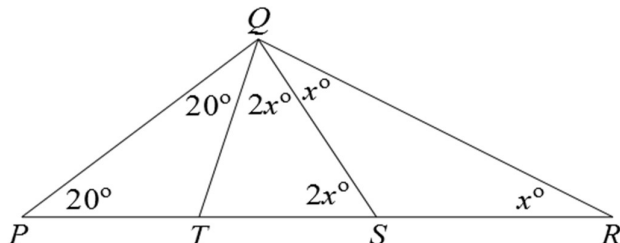
Since the interior angles of triangle PQR must sum to 180°, we obtain $20 + (20 + 2x + x) + x = 180$, and hence $x = 35$.

5. **Angle of overlap**
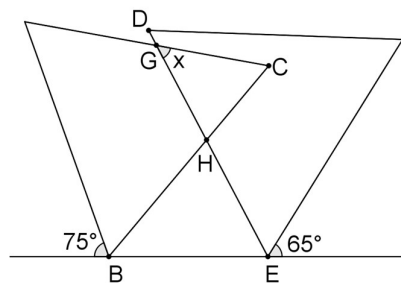
$\angle CBE = (180 - 75 - 60)° = 45°$

$\angle DEB = (180 - 65 - 60)° = 55°$

$\angle GHB = (45 + 55)° = 100°$ (exterior angle of a triangle)

$\angle HGC = (100 - 60)° = 40°$ (exterior angle of a triangle)

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